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AME-60639
Advanced Aerodynamics

Homework 4

I. Consider a circle of radius a centered at the origin. A source of strength Q is located at the point z_0. The method of images suggests that we place another source of the same strength inside the circle at the point $a^2/\bar{z}_0$. However, to satisfy the impermeability condition along the surface of the circle, we place a sink of the same strength at the origin. The combination of the two sources and sink gives the following complex conjugate velocity

$$u - iv = \frac{Q}{2\pi}\left(\frac{1}{z - z_0} + \frac{1}{z - a^2/\bar{z}_0} - \frac{1}{z}\right) \tag{1}$$

1. Calculate the velocity components perpendicular and tangential to the circle, v_r and v_θ and verify that $v_r = 0$.
2. Calculate and plot the pressure distribution along the circle.
3. Calculate the force applied to the circle due to the presence of the source. Does the source attracts or repel the circle?
4. If a vortex of circulation Γ is located at z_0. Find the expression for the velocity field and repeat 1 to 3.

II. A Joukowski airfoil has a chord length $c = 1m$, a thickness ratio $\theta = 0.06$ and a camber ratio $m = 0.04$. The airfoil profile can be determined from the Joukowski transformation which maps the region of the z plane outside of the circle of radius a centered at the point O' into the region outside the airfoil in the ζ plane.

$$\zeta = z + \frac{c_j^2}{z}, \tag{2}$$

where $z = x + iy$ and $c_j = OA$. The coordinates of the z plane are centered at the point O. The center O' of the circle is located at $\{-\epsilon_1, \epsilon_2\}$. Hence, along the circle

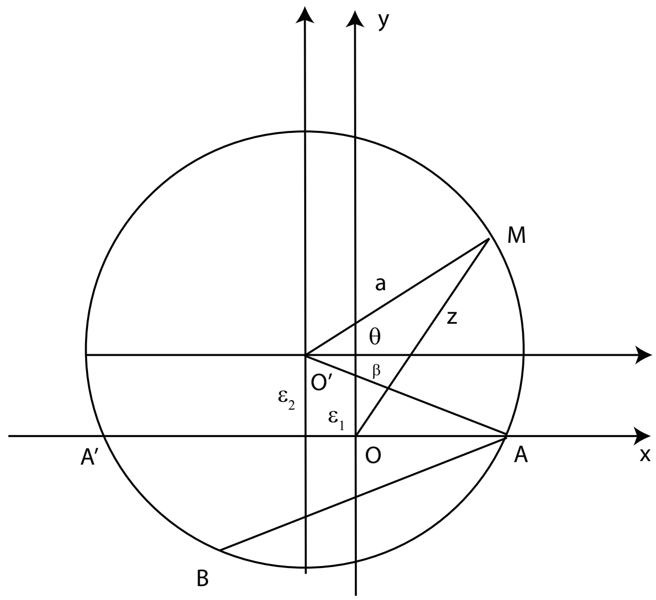

Figure 1: The Circle in the z Plane.

$$z = ae^{i\theta} - \epsilon_1 + i\epsilon_2. \tag{3}$$

For small thickness and camber ratios $\epsilon_1 << a$ and $\epsilon_2 << a$, The parameters of the Joukowski transformation, c_j, ϵ_1 and ϵ_2 can be approximately given in terms of the chord length,c, the thickness ratio, θ, and the the camber ratio, m,

$$c_j \approx \frac{c}{4}, \qquad \epsilon_1 \approx \frac{\theta c}{3\sqrt{3}} \qquad \epsilon_2 \approx \frac{mc}{2}. \tag{4}$$

1. Plot the profile of the Joukowski airfoil using (4) and compare the chord length, thickness and camber ratios to the exact values. What are the points of maximum thickness and camber of the Joukowski airfoil. Compare these results with those of a NACA airfoil having the same thickness and camber ratios and as well as the same location of maximum camber as the the Joukowski airfoil.
2. Calculate the velocity field and plot the pressure coefficient along the airfoil surface for both the Joukowski and NACA airfoils.

3. Calculate the surface pressure gradient for both airfoils and find the points where the pressure gradient becomes adverse, i.e., positive.
4. Calculate the lift for both airfoils by integrating the pressure along the surface and check the result with that obtained from the theorem of Blasius, i.e., $\vec{L} = \rho \vec{V} \times \vec{\Gamma}$.